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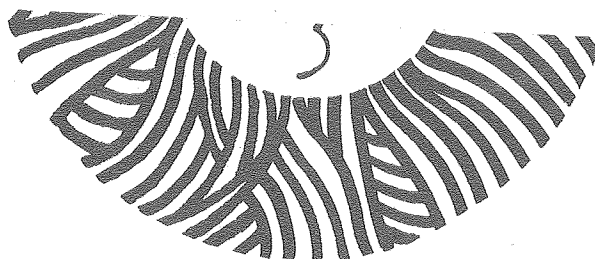
COMPARISON OF PROPORTIONAL AND ON/OFF COLLECTOR LOOP CONTROL STRATEGIES USING A DYNAMIC COLLECTOR MODEL

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COMPARISON OF PROPORTIONAL AND ON/OFF COLLECTOR LOOP CONTROL
STRATEGIES USING A DYNAMIC COLLECTOR MODEL

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ABSTRACT

Common control strategies used to regulate the flow of liquid through flat-plate solar collectors are discussed and evaluated using a dynamic collector model. Performance of all strategies is compared using different set points, flow rates, insolation levels and patterns (clear and cloudy days), and ambient temperature conditions.

The unique characteristic of the dynamic collector model is that it includes the effect of collector capacitance. In general, capacitance has a minimal effect on long term collector performance; however, short term temperature response and the energy-storage capability of the collector capacitance are shown to play significant roles in comparing on/off and proportional controllers. Inclusion of these effects has produced considerably more realistic simulations than any generated by steady-state models.

Simulations indicate relative advantages and disadvantages of both types of controllers, conditions under which each performs better, and the importance of pump cycling and controller set points on total energy collection.

Results show that the turn-on set point is not always a critical factor in energy collection since the collector stores energy while it is warming up and during cycling; and, that proportional flow controllers provide improved energy collection only during periods of interrupted or very low insolation. Although proportional controllers initiate flow at lower insolation levels than on/off controllers, proportional

controllers produce lower flow rates and higher average collector temperatures, resulting in slightly lower instantaneous collection efficiencies.

INTRODUCTION

Active solar heating systems are generally capital intensive; therefore, improvements which increase system efficiency must do so with only a small incremental initial cost in order for them to help solar energy compete with other energy sources. Since improved control strategies and controllers may satisfy these criteria, researchers and manufacturers have sought to evaluate and improve solar energy system controllers[5,8,9,11,12,13,15,16].

Commercially available controllers for domestic heating systems include both on/off and proportional control of the collector fluid[16]. While some manufacturers have advertised microprocessor based control systems, none of these systems are cost effective, as yet, for residential solar energy usage. On/off controllers have had the widest application due to their simplicity and generally reliable operation. However, demonstration projects [2,3,,6,14] have shown that two problems can occur with these controllers; 1) they can cause the circulating pump to cycle on and off excessively and 2) improper selection of set points can cause low system efficiency. In response to these problems some controller manufacturers have marketed proportional flow controllers claiming improved overall system efficiencies. This project was therefore undertaken to determine the relative merits of proportional and on/off control so that solar manufacturers and designers will be able to improve system efficiencies.

DYNAMIC FLAT-PLATE SOLAR COLLECTOR MODEL

The Hottel-Whillier-Bliss (H.W.B.) collector model [7], as adapted by Klein [10] to include the effects of capacitance, is used to describe the operation of a flat-plate solar collector. The model is based upon a heat balance on a tube and fluid element within a collector, where the entire capacitance of the collector is lumped within the tubes and the circulating fluid. The heat balance is solved using numerical methods on a digital computer to describe the circulating fluid's temperature as a function of time and space. The transient heat balance for a collector element of width W_c is:

$$\begin{aligned} \partial T_{f,x} / \partial t = & \gamma \left[(F'/C_A) [S - U_L(T_{f,x} - T_a)] - (\dot{m}c_p/C_A W_c) (\partial T_{f,x} / \partial x) \right] \\ & + (1 - \gamma) \left[(F'/C_A) [S - U_L(T_{f,x} - T_a)] \right] \end{aligned} \quad (1)$$

Where: If $\gamma = 1$ pump is running

If $\gamma = 0$ pump is not running

C_A is the weighted average of the total collector capacitance. This equation is for a non-drain down collector. For a drain down system a two lump model is required since the collector and fluid capacitance would have to be treated separately.

The spatial derivative is eliminated by breaking the collector into a number of stirred tanks; thus, the time dependent temperature of the Nth node is written:

$$\begin{aligned} dT_N/dt = & \gamma \left[(F'/C_A) [S - U_L(T_{f_N} - T_a)] + (\dot{m}c_p/C_A W_c \Delta x) (T_{f_{N-1}} - T_{f_N}) \right] \\ & + (1 - \gamma) \left[(F'/C_A) [S - U_L(T_{f_N} - T_a)] \right] \end{aligned} \quad (2)$$

This equation for 4 nodes was solved using the Parasol program [1] which

solves differential equations through the use of the fourth-order Runge-Kutta method. The Parasol program's output is the fluid temperature at different positions and for discrete time intervals.

The model described by equation 2 is adopted for the following reasons:

- 1) It provides a simple and accurate description of the transient temperature distribution in a collector's circulating fluid.
- 2) It included the effects of collector capacitance.
- 3) It is derived from a well established and respected collector model.
- 4) Results it provides are usable and consistent with known collector operation.

COLLECTOR PARAMETERS

To compare the various control strategies using a collector computer model, appropriate parameters must be used which represent a typical flat-plate collector under the influence of common external conditions. Although a multi-node model is used for the simulations, the single node model is used to define the parameters used. These parameters are then scaled for use in a multi-node model.

In the limiting case of a single node model of the collector, equation 2, for flow conditions, can be written to demonstrate the functional dependence of the collector temperature on 1) insolation and ambient temperature, 2) fluid flow rate and 3) collector characteristics:

$$C_A dT_{out}/dt = (K_{gain})f(t) + (K_{flow})T_{in} - (K'_{flow})T_{out}$$

Where:

K_{gain} represents the collector's gain from insolation and losses to the environment

$$K_{\text{gain}} = F'[S_{\text{max}} + U_L T_{a,\text{max}}]$$

$f(t)$ represents the time variation of the normalized forcing function due to insolation and ambient temperature

K_{flow} represents the fluid flow rate per unit area

$$K_{\text{flow}} = \dot{m}c_p/A_c$$

$$K'_{\text{flow}} = \dot{m}c_p/A_c + F'U_L ; \text{ since, } F'U_L \ll \dot{m}c_p/A_c$$

K_{flow} approximately equals K'_{flow}

C_A represents the collector/fluid capacitance per unit area

By allowing K_{gain} and K_{flow} (and K'_{flow}) to take on either HIGH or LOW values while keeping all other parameters constant, the various control strategies are compared based on a limited but comprehensive set of collector, meteorological, and flow variations which are used to define limits of operation for a typical collector. The numerical values for the parameters used are summarized in Table 1.

The dynamics associated with the storage tank and the piping are not considered to be critical for comparative results; therefore, the collector inlet temperature, T_{in} , is constant.

The solar day for all runs is 12 hours long with a peak insolation rate reached at hour 6. For modeling of a clear day (no interruptions of insolation) the insolation rate, I , is proportional to a sine wave with a 24 hour period. For a cloudy day (the view of the collector intermittently interrupted) the following equation, that was used by Close[4], determines the insolation rate as a function of time, t , in hours:

$$I = (I_{\max}/2)[\sin \pi t/12)][\cos(40 \pi t/12) + 1]$$

The ambient temperature, T_a , is proportional to a sine wave with a 24 hour period, the peak value is at the 9th hour of the solar day:

$$T_a = T_0 + T_M * \sin(\pi t/12 - \pi/4) \quad t = \text{hours}$$

COLLECTOR FLOW CONTROLLERS

The collection of solar energy is controlled by the flow of fluid through the collector loop. Collector outlet and storage tank temperatures are compared by a controller to determine the fluid flow rate. The difference between the collector outlet temperature and the storage tank temperature is known as ΔT and represents the temperature rise across the collector.

On/Off Flow Control

The on/off controller is a thermostat which turns the fluid circulation pump either on or off based on ΔT . The flow rate (\dot{m}) through the collector is defined by the following equations:

$$\dot{m} = \begin{cases} 0 & \left\{ \begin{array}{l} \text{if } \Delta T < \Delta T_{\text{on}} \text{ and last flow} = 0 \\ \text{or} \\ \text{if } \Delta T < \Delta T_{\text{off}} \end{array} \right. \\ \dot{m} & \left\{ \begin{array}{l} \text{if } \Delta T \geq \Delta T_{\text{on}} \\ \text{or} \\ \text{if } \Delta T \geq \Delta T_{\text{off}} \text{ and last flow} = \dot{m} \end{array} \right. \end{cases}$$

Where:

ΔT_{off} = temperature difference between fluid outlet and inlet sufficient to turn pump off.

ΔT_{on} = temperature difference between fluid outlet and inlet sufficient to turn pump on.

The region between ΔT_{on} and ΔT_{off} is known as the hysteresis zone. Because of hysteresis on/off controllers have "memory" which limits pump cycling.

Proportional Flow Control (with saturation)

In this type of feedback controller the fluid flow rate is varied as a function of the temperature rise across the collector, ΔT . The advantages of proportional controlled system are: fluid circulates at lower values of ΔT and pump cycling is minimized. The fluid flow rate through the collector can be described with the following equations:

$$\dot{m}(t) = \begin{cases} 0 & \text{for } \Delta T < \Delta T_{\text{off}} \\ K\Delta T & \text{for } \Delta T_{\text{off}} \leq \Delta T \leq \Delta T_{\text{max}} \\ \dot{m}_c & \text{for } \Delta T \geq \Delta T_{\text{max}} \end{cases}$$

Where:

\dot{m}_c = maximum flow rate

K = proportional flow constant equal to ratio of the maximum flow rate to the temperature difference required for maximum flow: $K = \dot{m}_c / \Delta T_{\text{max}}$

ΔT_{max} = temperature rise across collector at which flow rate saturates to its maximum

ΔT_{off} = the temperature rise across the collector sufficient to turn off the pump or the

minimum temperature rise across the
collector for which it is possible and/or
profitable to turn on the pump

DETERMINATION OF CONTROLLER SET POINTS

In determining proper controller set points there are two major considerations: set points must be chosen to maximize energy collection and minimize pumping power(or cost); and set points must be within the capability of the sensors used. The importance of sensor sensitivity and location cannot be overstressed since these two concerns have caused numerous problems in some solar installations.

The minimum temperature rise across the collector required for maintaining flow, ΔT_{off} , is the one that realizes an energy value collection rate equal to the energy cost of running the pump; therefore, ΔT_{off} can be shown to equal:

$$\frac{(\text{pumping power})(\text{pumping cost})(\text{heating system efficiency})}{(\text{fluid capacitance flow rate})(\text{heating cost})(\text{pump efficiency})}$$

This equation can be used for both on/off and proportional flow controllers. If a higher value of ΔT_{off} is used, say to meet sensitivity requirements of an uncalibrated sensor, less energy will be collected since the pump will turn on later, shut off sooner and cycle more than necessary.

Unlike ΔT_{off} , only a range of values can be determined for ΔT_{on} without knowledge of specific weather conditions. To determine an optimum range

for ΔT_{on} the steady-state H.W.B. model is used to analyse a solar collector. The maximum practical value for ΔT_{on} would be one that insures that the pump never cycles. That is, ΔT_{on} is set so that after the pump turns on at some level of absorbed insolation and ambient temperature the temperature rise across the collector does not fall below ΔT_{off} . Control stability requires that the minimum ΔT_{on} be greater than ΔT_{off} . Using these criteria it can be shown that the ratio of ΔT_{on} to ΔT_{off} should be greater than unity while less than the ratio of the capacitance flowrate to the approximate collector heat loss:

$$1 \leq \frac{\Delta T_{on}}{\Delta T_{off}} \leq \frac{\dot{m} c_p}{A_c F_R U_L}$$

For typical parameters $\Delta T_{on}/\Delta T_{off}$ is calculated to be less than thirty, much larger than typical ratios of 2 - 7 used in the solar industry[2,16] that provide satisfactory results while allowing some cycling at low temperatures or insolation levels.

CONTROLLER AND SET POINT COMPARISONS:

The controllers are compared on the basis of their performance with respect to: collection efficiency, maximum steady-state efficiency, pump running time and pump cycling. These comparisons are the results of digital computer simulations using a time step of 0.001 hours for high flow rates and 0.002 hours for low flow rates. The model is implemented on a PDP 11/60 computer.

A total of six controllers are compared under 8 different sets of conditions. The four on/off controllers have the following characteristics:

- A) $\Delta T_{\text{on}} = 5^{\circ}\text{C}(9^{\circ}\text{F})$, $\Delta T_{\text{off}} = 1.7^{\circ}\text{C}(3^{\circ}\text{F})$
- B) $\Delta T_{\text{on}} = 11.7^{\circ}\text{C}(21^{\circ}\text{F})$, $\Delta T_{\text{off}} = 1.7^{\circ}\text{C}$
- C) $\Delta T_{\text{on}} = 5^{\circ}\text{C}$ with a 'perfect' timer
- D) $\Delta T_{\text{on}} = 11.7^{\circ}\text{C}$ with a 'perfect' timer

The proportional controllers have the following characteristics:

- E) full flow at $\Delta T_c = 5^{\circ}\text{C} = \Delta T_{\text{max}}$, $\Delta T_{\text{off}} = 1.7^{\circ}\text{C}$
- F) full flow at $\Delta T_c = 11.7^{\circ}\text{C} = \Delta T_{\text{max}}$, $\Delta T_{\text{off}} = 1.7^{\circ}\text{C}$

The set points were picked to represent upper and lower limits of values used in industry and research. Timers are used to limit the amount of cycling; therefore, the 'perfect' timer eliminates all pump cycling.

One day simulations of different control strategies indicate how their operation varies with different set points, timers, meteorological conditions, and flow rates. Figure 1 shows a typical collector outlet temperature history generated by the model for a morning of low insolation. Table II presents the collection efficiencies, pump running times and amount of cycling for the different control strategies under the assigned conditions.

RESULTS

For the clear day cases, the collection efficiency for all but one of the controllers is approximately equal and not more than 7% below the maximum steady-state efficiency. The on/off controllers, in general, did slightly better with the on/off controllers with timers achieving the best efficiency since they run the pumps for the longest amount of time.

It is doubtful that any other type of controller could do better under similar conditions. During periods of interrupted insolation though, neither proportional nor on/off controllers respond well to rapid changes in the insolation rate and the collection efficiency falls well below the maximum possible.

Proportionally-controlled collectors can collect more energy during periods of interrupted insolation and/or very low insolation than on/off-controlled systems; because the proportional controller is more sensitive to changes in insolation and ambient temperature than the on/off controller. This sensitivity also causes the proportional controller to maintain a lower average flow rate and thus operate the collector at higher temperatures. While decreasing collection efficiency, this may improve storage stratification and overall system performance.

The on set point, ΔT_{on} , for an on/off controller can have a minimal effect on energy collection as long as it is not so high that the collector pump does not come on until late in the morning. This is because of the collector's capacitance, which allows the collector to store energy when the fluid is not circulating; energy which can be later released into the fluid. The fact that the collector acts as a storage device, also leads to the result that low to moderate cycling of the pump has a minimal effect on energy collection. The effects of collector capacitance are important and cannot be considered in steady-state analysis.

The proportional controller set point for maximum flow is found to have an effect on energy collection. If this point is too high, the flow rate will never reach maximum and thus losses to ambient are increased.

However, if the set point is too low, the proportional controller's sensitivity will be lost and the controller will act as a bang-bang controller.

The off set point for on/off and proportional control has simple criteria; that energy collected exceed parasitic pumping power and that the point selected meet sensor error requirements. The on set point, however, does not have simple criteria and can be defined only within a broad range.

Parasitic power required to run a circulating pump does not appear to be significant for either on/off or proportional controllers unless a large pump-motor is required, such as in a large drain down system.

CONCLUSIONS

The implications of this study for the design and evaluation of proportional and on/off control are two fold. First, the difference between a steady-state and a dynamic analysis of control strategies is significant. Future work in modeling control systems must consider collector capacitance in order to accurately describe the transient response of the fluid temperature. Second, neither on/off nor proportional control performs best for all conditions. Whether on/off or proportional control should be implemented is dependent on the weather conditions in the location being considered. It is hoped that the results of this analysis will be used as a guideline to indicate the general meteorological and flow rate conditions for which on/off or proportional control can be more advantageous.

Further work in the comparison of on/off and proportional control should include: 1) additional simulation studies using this or an improved dynamic solar system model which includes load loop dynamics, 2) experimental testing of the control strategies on facilities which can duplicate meteorological and load conditions for comparisons and 3) field tests.

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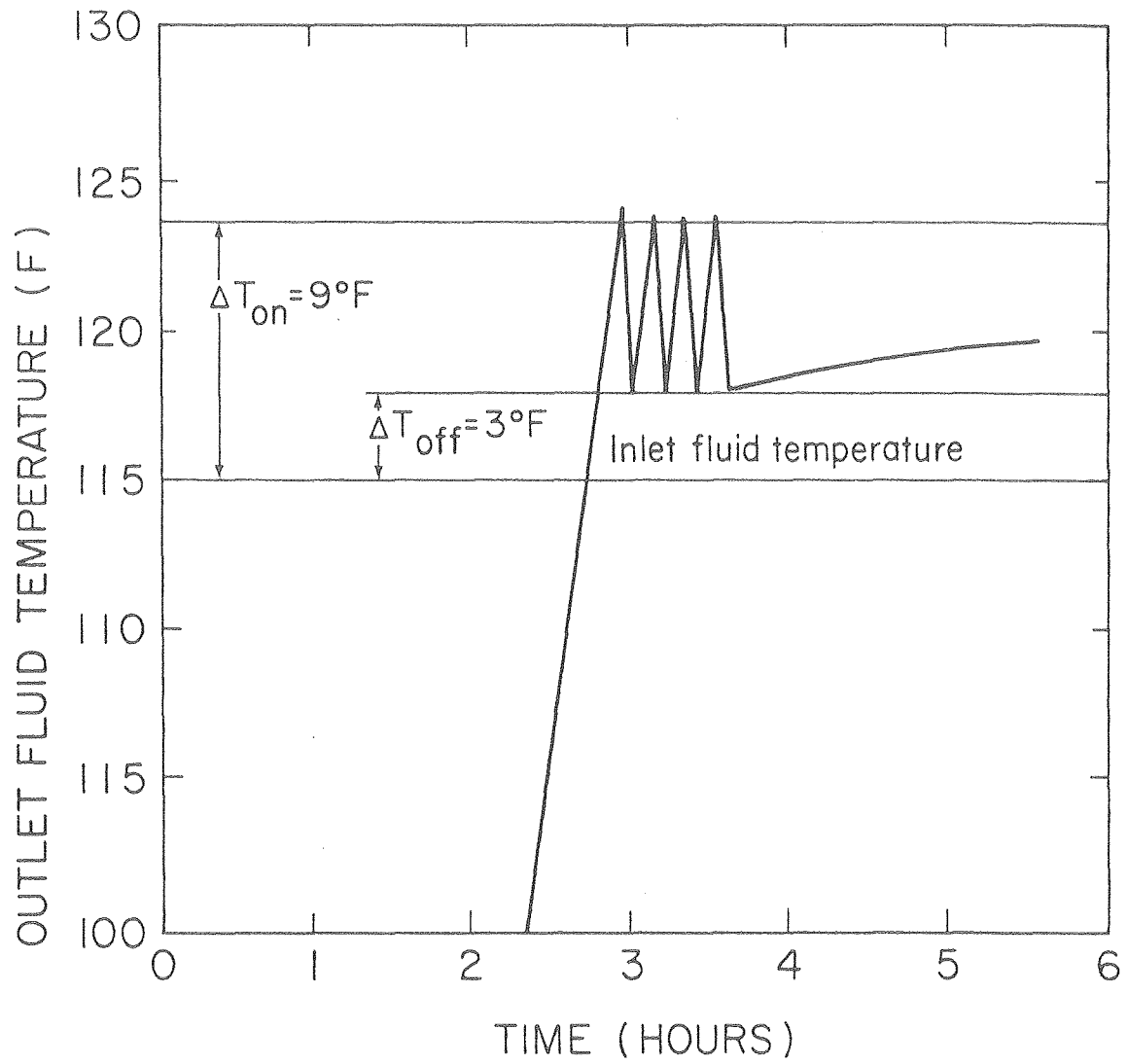


FIGURE 1: OUTLET FLUID TEMPERATURE
LOW GAIN, LOW FLOW, CLEAR DAY

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TABLE I: SUMMARY OF COLLECTOR PARAMETERS AND SIMULATION RUNS

<u>CAPACITANCE</u>	<u>HIGH GAIN</u>	<u>HIGH FLOW</u>
$C_A = .7 \text{ BTU/ft}^2\text{-}^\circ\text{F}$ $14.3 \text{ kJ/m}^2\text{-}^\circ\text{C}$	$I_{\max} = 300 \text{ BTU/ft}^2\text{-hr}$ 946 watts/m^2 $T_{a(\max)} = 70^\circ\text{F}$ 21.1°C	$\dot{m}c_p/A_c (\max) = 25 \text{ BTU/ft}^2\text{-hr-}^\circ\text{F}$ $511 \text{ kJ/m}^2\text{-hr-}^\circ\text{C}$
<u>COLLECTOR LOSS COEFFICIENT</u>	<u>LOW GAIN</u>	<u>LOW FLOW</u>
$U_L = .7 \text{ BTU/ft}^2\text{-hr-}^\circ\text{F}$ $3.97 \text{ watts/m}^2\text{-}^\circ\text{C}$	$I_{\max} = 150 \text{ BTU/ft}^2\text{-hr}$ 473 watts/m^2 $T_{a(\max)} = 50^\circ\text{F}$ 10°C	$\dot{m}c_p/A_c (\max) = 15 \text{ BTU/ft}^2\text{-hr-}^\circ\text{F}$ $306 \text{ kJ/m}^2\text{-hr-}^\circ\text{C}$
<u>TRANSMITTANCE/ABSORPTANCE</u>	<u>INLET FLUID TEMPERATURE</u>	<u>FIN EFFICIENCY</u>
$\tau\alpha = 0.84$	$T_{in} = 115^\circ\text{F}$ 46.1°C	$F' = .95 \text{ (flow)}$ 1.0 (no flow)

SUMMARY OF SIMULATION RUNS

Clear Day Runs $I = I_{\max}(\sin\pi t/12)$

<u>RUN #</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
GAIN:	HIGH	HIGH	LOW	LOW
FLOW:	HIGH	LOW	HIGH	LOW

Cloudy Day Runs $I = [I_{\max}/2][\sin(\pi t/12)][\cos(40\pi t/12) + 1]$

<u>RUN #</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
GAIN:	HIGH	HIGH	LOW	LOW
FLOW:	HIGH	LOW	HIGH	LOW

TABLE II: CONTROLLER STRATEGY COMPARISONS

12 Hour Totals

CONTROL STRATEGY		HIGH GAIN ^a HIGH FLOW ^b CLEAR DAY	HIGH GAIN LOW FLOW ^c CLEAR DAY	LOW GAIN ^d HIGH FLOW CLEAR DAY	LOW GAIN LOW FLOW CLEAR DAY	HIGH GAIN HIGH FLOW CLOUDY DAY ^e	HIGH GAIN LOW FLOW CLOUDY DAY	LOW GAIN HIGH FLOW CLOUDY DAY	LOW GAIN LOW FLOW CLOUDY DAY
Maximum Steady-State Efficiency(%)		65.7	65.7	39.5	39.5	56.1	56.1	26.5	26.5
ON/OFF On=9°F (5°C) Off=3°F (1.7°C)	efficiency(%)	60.3	59.6	35.0	34.9	45.2	45.2	8.6	8.5
	pumping time(hours)	8.72	9.27	2.76	5.98	3.34	3.83	.311	.496
	times cycled	10	2	61	10	14	12	4	10
ON/OFF On=21°F (11.7°C) Off=3°F (1.7°C)	efficiency(%)	59.7	59.1	31.9	33.9	44.1	44.2	5.2	5.4
	pumping time(hours)	8.39	8.98	1.39	5.44	2.47	2.92	0.095	0.16
	times cycled	6	2	22	6	12	18	2	2
ON/OFF With perfect timer On=9°F 5°C	efficiency(%)	60.5	59.9	35.7	35.3	--	--	--	--
	pumping time (hours)	9.87	9.88	7.68	7.69	--	--	--	--
	times cycled	0	0	0	0	--	--	--	--
ON/OFF With perfect timer On=21°F 11.7°C	efficiency(%)	60.4	59.8	35.5	35.1	--	--	--	--
	pumping time(hours)	9.71	9.72	7.38	7.39	--	--	--	--
	times cycled	0	0	0	0	--	--	--	--
PROPORTIONAL Full On=9°F 5°C Off = 3°F 1.7°C	efficiency(%)	60.2	59.7	35.0	34.7	45.4	45.0	9.6	9.5
	pumping time (equiv. hours)	7.54	8.85	3.58	4.63	3.20	4.03	0.52	0.72
	times cycled	0	0	0	0	0	0	0	0
PROPORTIONAL Full On=21°F 11.7°C Off = 3°F 1.7°C	efficiency(%)	59.6	59.0	34.4	33.9	44.8	44.3	9.4	9.1
	pumping time (equiv. hours)	4.92	6.33	2.34	3.01	2.16	2.84	0.38	0.51
	times cycled	0	0	0	0	0	0	0	0

a) high gain: insolation = 2292 BTU/ft²-day
7224 watt-hrs/m²-day
ambient temp. = 44.4°F - 70°F
6.89°C - 21.1°C

b) high flow = 25 lbm/hr-ft²
122 kg/hr-m²

c) low flow = 15 lbm/hr-ft²
73.2 kg/hr-m²
d) low gain: insolation=1146 BTU/ft²-day
3612 watt-hrs/m²-day
ambient temp.= 32.9°F - 50°F
.5°C - 10°C

inlet temperature = 115°F
46.1°C

collector capacitance = .7 BTU/ft²-°F
14.3 kJ/m²-°C

collector loss coefficient = .7 BTU/ft²-hr-°F
3.97 watts/m²-°C

e) for cloudy day cases, the total insolation is half of the clear day values given in (a) and (d)

NOMENCLATURE

C_a	Effective value of collector capacitance, per unit collector area
c_p	Thermal capacitance of circulating fluid
F'	Plate fin efficiency factor
K	Proportionality constant for proportional controllers
K_{flow}	Represents the fluid flow rate, per unit area
K_{gain}	Represents the collector's gain from insolation and losses to the environment, per unit area
\dot{m}	Fluid mass flow rate
S	Rate of absorption of solar insolation by collector plate, per unit area
t	Time
T_a	Ambient temperature
T_M	Ambient temperature calculation constant
T_O	Ambient temperature calculation constant
$T_{f,x}$	Fluid temperature at position x
T_{in}	Inlet fluid temperature
T_{out}	Outlet fluid temperature
U_L	Collector loss coefficient, per unit area
W_c	Width of collector in the direction of flow
x	Displacement in flow direction
	Pump control indicator
ΔT_{max}	Temperature across collector at which flow rate saturates to its maximum, for proportional control
ΔT_{off}	Temperature rise across the collector sufficient to turn off the pump
ΔT_{on}	Temperature rise across the collector sufficient to turn on the pump

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March Manufacturing Inc., Glenview, IL.

Natural Power, Inc., New Boston, NH.

Piper Hydro Inc., Anaheim, CA.

PPG Industries Glass Division, Pittsburg, PA.

Rho Sigma Inc., Van Nuys, CA;

Solar Control Corp., Boulder, CO;